

# DETERMINISM AND THE GRAVITATIONAL PLANCK CONSTANT.

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## Abstract

A discussion is given of the uncertainty principle in view of the introduction of a Gravitational Planck Constant. The need for such a gravitational constant is shown first. A reduced electromagnetic Planck constant and the analogous reduced gravitational Planck constant are defined as  $\hbar/e^2$  and  $H/m^2$  respectively. An attempt is made to reconcile the quantum uncertainty concepts with a deterministic view of the physical world. This conclusion is achieved through the detailed analysis of the measurement procedures of physical quantities.

**Key words:** uncertainty principle, quantum gravity, gravitation, Planck constant, determinism.

# 1 INTRODUCTION.

A single value of the Planck constant has been used until now when treating both electromagnetic and gravitational phenomena. This is a reasonable assumption since most of the gravitational experiments we can think about are performed using simultaneously electromagnetism and gravitation: all our detectors are electromagnetic devices. Nevertheless it is easy to think about phenomena where electromagnetism does not play a role. It has been proposed that when dealing with *purely gravitational* phenomena, the Planck constant  $h$  should be replaced with a gravitational counterpart ( $H$  from now on) [1, 2].  $H$  has been defined as [1]:

$$H = \frac{1}{8\pi \varepsilon_0 G} \times \frac{m^2}{e^2} \times h \quad (1)$$

with a value:  $H = 7.95 \times 10^{-77}$  J s. Here  $e$  and  $m$  are the electron charge and mass and therefore this constant relates to the electron. This aspect will be discussed later.

## 2 THE NEED FOR A GRAVITATIONAL PLANCK CONSTANT.

In order to evidence the need for a Gravitational Planck Constant, we will start with a discussion of the emission mechanism from an electromagnetic dipole, a very well understood case. In such a system the quantum theory holds that photons are emitted with an energy  $E = h\nu$ . A 1000 MHz microwave antenna with a RF power of 10 W is supposed to emit some  $10^{25}$  photons per second. Each photon is admitted to show a particle-like behaviour, while the ensemble behaviour of these photons gives rise to the wave aspect of their propagation. The particle-like behaviour of each photon is not accessible to us in this case, only because of the technological limits of our detectors [3]. We observe waves, but definitely nobody would conclude that the emission is not quantized as photons. The number of photons emitted per second can be easily evaluated starting from realistic numbers: for instance a 10 W electronics creating a 1 A RF current from a 10 V supply. In this case some  $6 \times 10^{18}$  electrons are accelerated through a 10 V potential difference every second. We can divide the 10 eV associated energy drop into steps of size  $h\nu/e$  ( $\simeq 4 \times 10^{-6}$  V at 1000 Mhz), getting  $2.5 \times 10^6$  such "energy levels". It is within the quantum ideas to think of a photon being created every time one electron passes such steps. We end up with  $\simeq 10^{25}$  photons per second. This calculation is obviously tautological, but has the purpose of evidencing the physical process.

We do not expect gravitational (dipole) radiation to arise from such a process.

Nevertheless, starting from the above example, we can easily accept that in more complex systems (like an exploding supernova or a pulsar) the emitting agents are the single elementary particles involved in the macroscopic process - anytime a quadrupole change in the gravitational field acts on them. This view has been proposed earlier: see for instance Ref. [4]. The correct interpretation of quantum ideas suggests that a large number of gravitational quanta are emitted: in analogy with the RF dipole example we expect to observe a gravitational wave. Being the gravitational field more than 40 orders of magnitude weaker than the electromagnetic field involved during the same processes, we expect that the gravitational wave carries an energy some 40 orders of magnitude smaller than the electromagnetic one. If we want to keep the energy conservation law, we have two choices:

1. hypothesize that the emission probability of each elementary emitter is more than 40 orders of magnitude smaller for gravitational than for electromagnetic emission (see for instance Zeldovich and Novikov [4]); or
2. hypothesize that the energy carried by each graviton is more than 40 orders of magnitude smaller than the energy of the corresponding photon, the emission probability being of the same order of magnitude.

The first possibility is rather queer, and it does not comply with the above description of the emission process; in addition, it has never been proved. The second possibility calls for the use of a gravitational Planck constant as defined in Eq. (1).

According to such definition, this constant depends on the mass of the test particles involved in the gravitational process. This peculiarity is hardly surprising if we consider that charge takes only one value as it does the electromagnetic Planck constant  $h$ , while particles appear with a spectrum of masses. Therefore, the use of a mass-dependent gravitational constant is the natural extension of the single valued electromagnetic one.

### 3 THE REDUCED PLANCK CONSTANTS.

Nevertheless it has a meaning the definition of a "reduced gravitational Planck constant":

$$H^* = H/m^2 \quad (2)$$

which is independent on the mass of the emitting system and the analogous "reduced electromagnetic Planck constant":

$$h^* = h/e^2 \quad (3)$$

with numerical values:

$$\begin{aligned} H^* &= 9.58 \times 10^{-15} \text{ J s/kg}^2 \\ h^* &= 2.58 \times 10^4 \text{ J s/coul}^2 \end{aligned}$$

We note here that the reduced electromagnetic Planck constant enters the equations governing several physical phenomena (the Bohr formula, the fine structure constant, the relativistic correction to the hydrogen atom energy levels, the quantum Hall effect - for example). Alfonso-Faus has proposed a new system of units such that Local Lorentz Invariance and Local Position Invariance is preserved [5]. In this system  $\varepsilon_0 = \mu_0 = 1/c$  and the fine structure constant turns out to be the inverse of the reduced electromagnetic Planck constant:

$$\alpha = \frac{e^2}{2h} = \frac{1}{2h^*}$$

The reduced electromagnetic Planck constant can be explicated also in the relation

$$E = h\nu = h^* \times e^2 \times \nu \quad (4)$$

This writing has the advantage of showing explicitly the dependence of the energy of the electromagnetic quantum on the value of the emitting charge, besides the well known dependence on the frequency. The charge dependence is usually ignored because at the microscopic level charge takes a single value in most cases.

Coming back to the multiple valuedness of the gravitational Planck constant (Eq. 1), we can compare Eq. (4) with the gravitational analogue:

$$H = H\nu = H^* \times m^2 \times \nu \quad (5)$$

We see that this multiple valuedness implies that different particles undergoing the same energetic transition will emit gravitational quanta with different frequencies.

## 4 DETERMINISM OR UNCERTAINTY?

The introduction of a gravitational Planck constant prompts the idea of "gravitational uncertainty relations" [1, 2]. For instance, in *purely gravitational* phenomena the momentum uncertainty relation should be written:

$$\Delta x \times \Delta p_x = \frac{H}{2\pi} \quad (6)$$

We would be faced now with two levels of uncertainty: the "coarse" electromagnetic level and a much finer gravitational level. Are the two in contraddiction? Do they pose unsolvable inconsistencies? Can we violate the limits of the electromagnetic uncertainty by taking advantage of the gravitational one? In this paragraph we will give a tentative answer.

Any observation of microscopic phenomena, must use a suitable probe. A careful analysis of the foundations of our experimental techniques shows that our probes all satisfy two requirements. First, the energy of the probe is always of (it cannot be other than) the same order of magnitude of the typical energies involved in the system under observation. Second, all probes are - directly or indirectly - electromagnetic in their nature. These statements are obvious in a sense, since anybody knows implicitly that they are true; nevertheless, at our knowledge, they have not been stressed in the analysis of the uncertainty concepts. The consequence of the mentioned requirements is that as soon as we apply our probe to the measured system, we perturb it - as it is well known. This is different from our macroscopic experience, where we can choose the probe such as to reduce the perturbation (the "measurement error") to an acceptable (say sub-one-percent) level. At the microscopic level, if we prepare a particle to travel in a certain direction, it will continue to travel in the same direction with the same momentum, unless we shoot it with a probe. This is guaranteed by the inertia principle, which is believed to be valid at all levels in physics. In an experimental set-up aimed at the test of Eq. (6), in order to give a measurement result we shall prepare a large number of target particles - a beam - and we shall shoot them with a large number of probe particles: we need such large numbers to achieve a statistically significant figure as the result of our experiment. After the interaction, we will observe a distribution of momenta for the outgoing target particles. This distribution reflects the distribution of interaction channels which have been sampled by different target-probe couples. We conclude that the probabilistic behaviour is not intrinsic to the observed system, but is caused by our measurement protocol and by the rudeness of our probes. The simplest and plainest description of such a measurement process requires that the behaviour of the target particles is deterministic, while the results of our observation are (cannot be other than) probabilistic.

## 5 CONCLUSIONS

Coming back to the gravitational uncertainty relations, it is obvious that *purely* gravitational probes offer in principle a reduced perturbation on the measured system. The smaller intensity of the force gives a support to the use of the gravitational Planck constant. A much smaller uncertainty could be possible in our measurements, provided we could get the technology for the production and detection of single gravitons. Lacking such devices and using instead gravitational wave detectors we would be very much in the situation of the 19th century wave optics: no need for quantization. On a different point of view, single graviton effects would be buried by far in the electromagnetic thermal noise in all practical situations. From the conceptual point of view the introduction of the gravitational relations will not create a new limit of uncertainty; based on the above analysis, the physical world remains deterministic: only the results of our measurements would be represented with different statistical distributions.

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